QUANTITATIVE METHODS

Continuous Uniform Distributions

Outcomes can only be between *a* and *b*; the probability of an outcome outside the boundaries of *a* and *b* is zero.

Cumulative density function (cdf) for a uniform distribution :

$F(x) = 0$ for $x \leq a$

$F(x) = (x-a)/(b-a)$ for $a < x < b$

$F(x) = 1$ for $x \geq b$

Binomial Distribution

Outcome can either be "success" or "failure."

$$p(x) = \frac{n!}{(n-x)!x!} p^x (1-p)^{n-x}$$

Binomial Random Variable

$E(X) = np$

$Var(X) = np(1-p) = npq$

Standardized Random Variables

A *standardized random variable* is normalized so that it has a mean of zero and a standard deviation of 1.

Z-score : represents # of standard deviations a given observation is from a population mean.

$$z = \frac{\text{observation} - \text{population mean}}{\text{standard deviation}} = \frac{x-\mu}{\sigma}$$

Correlation Coefficient, *r*

The *correlation coefficient* is bounded by –1 and +1. If the correlation is zero, the variables *x* and *y* are uncorrelated.

$$r_{x,y} = \frac{\text{cov}(X,Y)}{\sigma_x \sigma_y} = \sqrt{R^2}$$

Measures of Central Tendency

Arithmetic mean : sum of all observation values in sample/population divided by the number of observations.

Mode : value that occurs most frequently in a data set.

Median : midpoint of a data set when data is arranged in an ascending or descending order.

Variance and Standard Deviation

Variance : average of squared deviations from the mean.

$$\text{population variance} = \sigma^2 = \frac{\sum_{i=1}^{N}(X_i - \mu)^2}{N}$$

$$\text{sample variance} = s^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$$

Standard deviation : square root of the variance.

Normal Distribution

Normal distribution is completely described by its mean and variance.

- 68% of observations fall within ± 1σ.
- 90% of observations fall within ± 1.65 σ.
- 95% of observations fall within ± 1.96 σ.
- 99% of observations fall within ± 2.58 σ.

Skewness

- *Positively skewed* : mean > median > mode.
- *Negatively skewed* : mean < median < mode.

Kurtosis

- *Leptokurtic* : more peaked than a normal distribution (fat tails); excess kurtosis > 0.
- *Platykurtic* : flatter than a normal distribution; excess kurtosis < 0.

Standard Error

Standard error of the sample mean is the standard deviation of the distribution of sample means.

$$\text{known population variance} = \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$\text{unknown population variance} = s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

Confidence Intervals

The *confidence interval* gives the range of values the mean value will be between, with a certain probability (e.g., 90% or 95%). With known variance, the formula for a confidence interval is:

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$z_{\alpha/2} = 1.65$ for 90% confidence intervals (significance level 10%, 5% in each tail)

$z_{\alpha/2} = 1.96$ for 95% confidence intervals (significance level 5%, 2.5% in each tail)

$z_{\alpha/2} = 2.58$ for 99% confidence intervals (significance level 1%, 0.5% in each tail)

Hypothesis Testing

Null hypothesis (H_0) : hypothesis the researcher wants to reject; hypothesis that is actually tested; the basis for selection of the test statistics.

Alternative hypothesis (H_a) : what is concluded if there is significant evidence to reject the null hypothesis.

One-tailed test : tests whether value is greater than or less than 0.

$H_0: \mu \leq 0$ versus $H_a: \mu > 0$

Two-tailed test : tests whether value is different from zero.

$H_0: \mu = 0$ versus $H_a: \mu \neq 0$

Type I error : rejection of null hypothesis when it is actually true.

Type II error : failure to reject the null hypothesis when it is actually false.

Simple Linear Regression

General form of linear regression model:

$Y_i = b_0 + b_1 X_i + \varepsilon_i$

- Y_i = dependent variable; estimated value of Y_i, given X_i.
- X_i = independent variable.
- b_0 = intercept term; represents value of Y if X is zero.
- b_1 = slope coefficient; measures change in Y for a one-unit change in X.

$$\hat{b}_1 = \frac{\text{cov}(X,Y)}{\text{variance}(X)}$$

Regression Coefficients: Testing Significance

Appropriate test structure :

$H_0: b_1 = 0$ versus $H_a: b_1 \neq 0$

$$\text{Test: } t_{b_1} = \frac{\hat{b}_1 - b_1}{s_{\hat{b}_1}}$$

Decision rule:

reject H_0 if $t > +t_{critical}$ or if $t < -t_{critical}$

Coefficient of Determination, R^2

Percentage of the total variation in Y explained by X.

$$R^2 = \left(\frac{SSR}{SST}\right) = 1 - \left(\frac{SSE}{SST}\right) = \frac{\text{explained variation}}{\text{total variation}}$$

VALUE AT RISK

Value at Risk (VAR)

Minimum amount one could expect to lose with a given probability over a specific period of time.

$VAR(X\%) = z_{x\%} \times \sigma$

Use the square root of time to change daily to monthly or annual VAR.

$VAR(X\%)_{1-days} = VAR(X\%)_{1-day} \sqrt{1}$

Factors that Affect Portfolio Risk

- Asset concentration.
- Asset volatility.
- Asset correlation.
- Systematic risk.

Linear vs. Nonlinear Derivatives

- A derivative is described as *linear* when the relationship between an underlying factor and the derivative's value are linear in nature.
- A *nonlinear* derivative's value is a function of the change in the value of the underlying asset and is dependent on the state of the underlying asset.

...CONTINUED

Cash Flow at Risk

Measures the cash flow shortfall; useful for firms such as manufacturers that have non-traded assets and rely on internally generated cash to grow.

$CFAR = \alpha \times$ volatility of cash flow

Costs of CFAR should be incorporated into NPV calculations.

Portfolio VAR

Diversified VAR :

$$VAR_p = Z_c P \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{1,2}\sigma_1\sigma_2}$$

$$VAR_p = \sqrt{VAR_1^2 + VAR_2^2}$$

Undiversified VAR :

$$VAR_p = \sqrt{VAR_1^2 + VAR_2^2 + 2VAR_1VAR_2} = VAR_1 + VAR_2$$

Marginal VAR is the per dollar change in a portfolio VAR that occurs from an additional investment in a position.

Incremental VAR is the change in VAR from the addition of a new position in a portfolio.

Component VAR is the amount a portfolio VAR would change from deleting a position in a portfolio.

$$CVAR_i = (MVAR_i) \times (w_i \times P) = VAR \times \beta_i \times w_i$$

Exponentially Weighted Moving Average Model

$$\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1-\lambda)r_{n-1}^2$$

where λ = weight on previous volatility estimate (λ between zero and 1).

RiskMetrics® and GARCH models are both exponential smoothing weighting methods.

GARCH Estimation Model

$$\sigma_n^2 = \omega + \alpha r_{n-1}^2 + \beta\sigma_{n-1}^2$$

- Implicitly assumes variance reverts to a long-run average level.
- Sum of (α + β) must be less than one for the model to be stationary.

Approaches for Estimating VAR

- Historical-based: parametric, nonparametric, and hybrid.
- Implied-volatility-based: uses derivative pricing models.

Stress Testing

VAR tells the probability of exceeding a given loss but fails to incorporate the possible amount of a loss that results from an extreme amount.

Stress testing complements VAR by providing information about the magnitude of losses that may occur in extreme market conditions.

Risk Budgeting

VAR is useful for the investment process.

For a choice between two new positions for a portfolio, compare the marginal VARs to make the selection. When deciding whether to increase one existing position over another, compare the return-to-VAR ratios and increase the position in the one with the higher ratio.

Budgeting risk across asset classes : selecting assets whose combined VARs are less than the total allowed.

Budgeting risk across active managers : the optimal allocation is achieved with the following formula:

weight of portfolio managed by manager i

$$IR_i \times \left\{\frac{\text{portfolio's tracking}}{\text{error volatility}}\right\} = IR_p \times \left\{\frac{\text{manager's tracking}}{\text{error volatility}}\right\}$$

MARKET RISK MANAGEMENT

Foreign Currency Risk

A net long (short) currency position means a bank faces the risk that the FX rate will fall (rise) versus the domestic currency.

On-balance sheet hedging : matched maturity and currency foreign asset-liability book.

Off-balance sheet hedging : enter into a position in a forward contract.

DERIVATIVES

Forward Prices:

Basic Formula: $F_0 = S_0e^{rT}$

Forward price with carrying costs:

$F_0 = (S_0 - I)e^{rT}$

Forward price with continuous dividend yield(q):

$F_0 = S_0e^{(r-q)T}$

Forward price with storage costs:

$F_{0,T} \geq S_0e^{rT} + \lambda(0,T)$ or $F_{0,T} = S_0e^{(r+\lambda)T}$

Arbitrage : Remember buy low, sell high!

- If $F_0 > S_0e^{rT}$, borrow, buy spot, sell forward today; deliver asset, repay loan at end.
- If $F_0 < S_0e^{rT}$, short spot, invest, buy forward today; collect loan, buy asset under futures contract, deliver to cover short sale.

FIXED INCOME SECURITIES

Relationship Among Coupon, YTM, and Price

If coupon rate > YTM, bond price will be greater than par value: *premium bond.*

If coupon rate < YTM, bond price will be less than par value: *discount bond.*

If coupon rate = YTM, bond price will be equal to par value: *par bond.*

Clean and Dirty Prices

Clean price : bond price without accrued interest.

Dirty price : includes accrued interest; price the seller of the bond must be paid to give up ownership.

Equivalent Annual Yield

$$EAY = \left[1 + \frac{r}{n}\right]^n - 1$$

Duration and Convexity

Duration : first derivative of the price/yield relationship; most widely used measure of bond price volatility; the longer (shorter) duration, the more (less) sensitive the bond's price to changes in interest rates; can be used for linear estimates of bond price changes.

$$\text{effective duration} = \frac{BV_{-\Delta y} - BV_{+\Delta y}}{2 \times BV_0 \times \Delta y}$$

Convexity : measure of the degree of curvature (second derivative) of the price/yield relationship; accounts for error in price change esrimates from duration.

$$\text{convexity} = \frac{BV_{-\Delta y} + BV_{+\Delta y} - 2 \times BV_0}{BV_0 \times \Delta y^2}$$

Percentage price change estimate using duration and convexity :

$$[-\text{duration} \times \Delta y \times 100] + \left[\left(\tfrac{1}{2}\right) \times \text{convexity} \times (\Delta y)^2 \times 100\right]$$

Macaulay and Modified Duration

Modified duration : approximate percentage price change in a bond from a given change in yield.

Macaulay duration : weighted average term to maturity of a bond's cash flows.

$$\text{Macaulay duration} = \left(1 + \frac{y}{2}\right) \times \text{modified duration}$$

- The Macaulay duration of a zero coupon bond is equal to its maturity.

Characteristics of Duration

- Maturity increases, duration increases.
- Coupon increases, duration decreases.
- Yield decreases (price increases), duration increases.
- Yield increases (price decreases), duration decreases.

Bond Portfolio Structures

Barbell : manager uses bonds with short and long maturities.

Bullet : manager buys bonds concentrated in the intermediate maturity range.

If a bullet and barbell have the same duration, the barbell portfolio will have greater convexity because convexity is related to the square of maturity.

Key Rate Duration

- Duration assumes a parallel shift in the yield curve.
- Key rate duration addresses nonparallel shifts in the yield curve by allowing for changes in all rates to be determined by changes in key rates.

Hedging Strategies Using Futures

Basis risk : difference between spot price of hedged asset and futures price of contract used in a hedge

Optimal (minimum variance) hedge ratio:

$$h = \rho_{S,F}\frac{\sigma_S}{\sigma_F}$$

Day Count Conventions

- *Actual/Actual.* T-bonds.
- *30/360.* U.S. Corporate and municipal bonds.
- *Actual/360.* T-bills and other money market instruments.

Interest Rate Swaps

Plain vanilla interest rate swap: exchanges fixed for floating-rate payments over the life of the swap.

At inception, the value of a swap is zero.

After inception, the value of the swap is the difference between the present value of the remaining fixed- and floating-rate payments:

$$V_{\text{swap to pay fixed}} = B_{\text{float}} - B_{\text{fix}}$$

$$V_{\text{swap to receive fixed}} = B_{\text{fix}} - B_{\text{float}}$$

$$B_{\text{fixed}} = \left(PMT_{\text{fixed},t_1} \times e^{-(rt_1)}\right) + \left(PMT_{\text{fixed},t_2} \times e^{-(rt_2)}\right) + \ldots + \left[\left(\text{notional} + PMT_{\text{fixed},t_n}\right) \times e^{-(rt_n)}\right]$$

$$B_{\text{floating}} = \left[\left(\text{notional} + \text{notional} \times \frac{r_{\text{floating}}}{2}\right) \times e^{-(rt_1)}\right]$$

Currency swap : exchanges payments in two different currencies; payments can be fixed or floating.

If a swap has a positive value to one counterparty, that party is exposed to credit risk.

Factors Affecting an Option's Price

Factor	European call	European put	American call	American put
S	+	–	+	–
X	–	+	–	+
T	?	?	+	+
σ	+	+	+	+
r	+	–	+	–
D	–	+	–	+

Option Pricing Bounds

Upper bound European/American call:

$$c \le S_0 \; ; \; C \le S_0$$

Upper bound European/American put:

$$p \le Xe^{-rT} \; ; \; P \le X$$

Lower bound European call on non-dividend-paying stock:

$$c \ge \max\left(S_0 - Xe^{-rT}, 0\right)$$

Lower bound European put on non-dividend-paying stock:

$$p \ge \max\left(Xe^{-rT} - S_0, 0\right)$$

Rules for Exercising American Options

- It is never optimal to exercise an American call on a non-dividend-paying stock before its expiration date.
- American puts can be optimally exercised early if they are sufficiently in-the-money.
- An American call on a dividend-paying stock may be exercised early if the dividend exceeds the amount of forgone interest.

Put-Call Parity

$$P + S_0 = C + Xe^{-rT}$$

Binomial Option Pricing

Step 1: Calculate option payoffs at end in all states.

Step 2: Calculate option values using risk-neutral probabilities.

$$\text{size of up move} = U = e^{\sigma\sqrt{t}}$$

$$\text{size of down move} = D = \frac{1}{U}$$

$$\pi_{up} = \frac{e^{rt} - D}{U - D}; \; \pi_{down} = 1 - \pi_{up}$$

Step 3: Discount to today using risk-free rate.

Black-Scholes-Merton Option Pricing Model

$$c = S_0 N(d_1) - Xe^{-rT} N(d_2)$$

$$p = Xe^{-rT}\left[1 - N(d_2)\right] - S_0\left[1 - N(d_1)\right]$$

Volatility Smiles

Currency options : implied volatility is lower for at-the-money options than it is for away-from-the-money options.

Equity options : higher implied volatility for low strike price options.

Option Trading Strategies

- *Covered call.* Long stock plus short call.
- *Protective put.* Long stock plus long put. Also called portfolio insurance.
- *Bull spread.* Purchase call option with low exercise price, and subsidize the purchase with sale of a call option with a higher exercise price.
- *Bear spread.* Short bull spread. Purchases call with high strike price and shorts call with low strike price. Investor keeps difference in price of the options if stock price falls. Bear spread with puts involves buying put with high exercise price and selling put with low exercise price.
- *Butterfly spread.* Three different options. Buy one call with low exercise price, buy another with a high exercise price, and short two calls with an exercise price in between. Butterfly buyer is betting the stock price will stay near the price of the written calls.
- *Calendar spread.* Two options with different expirations. Sell a short-dated option and buy a long-dated option. Investor profits if stock price stays in a narrow range.
- *Long straddle.* Bet on volatility. Buy a call and a put with the same exercise price and expiration date. Profit is earned if stock price has a large change in either direction.
- *Short straddle.* Sell a put and a call with the same exercise price and expiration date. If stock price remains unchanged, seller keeps option premiums. Unlimited potential losses.
- *Strangle.* Similar to straddle, except purchased option is out-of-the money, so it is cheaper to implement. Stock price has to move more to be profitable.
- *Strips and straps.* Add an additional put (strip) or call (strap) to a straddle strategy.

Greeks

Delta : estimates the change in value for an option for a 1-unit change in stock price.

- Call delta between 0 and +1; increases as stock price increases.
- Call delta close to 0 for far out-of-the-money calls; close to 1 for deep in-the-money calls.
- Put delta between –1 and 0; increases from –1 to 0 as stock price increases.
- Put delta close to 0 for far out-of-the-money puts; close to –1 for deep in-the-money puts.

Theta : time decay; most negative when option is at-the-money and close to expiration.

Gamma : rate of change in delta as underlying stock price changes; largest when option is at-the-money.

Vega : sensitivity of an option's price to changes in volatility; largest when option is at-the-

- Changes in each key rate will affect the term of the previous and subsequent key rates in a linear fashion.

Bonds With Embedded Options

Callable bond : issuer has the right to buy back the bond in the future at a set price; as yields fall, bond is likely to be called; prices will rise at a decreasing rate — *negative convexity.*

Putable bond : bondholder has the right to sell bond back to the issuer at a set price.

MBS Prepayment Risk

Factors that affect prepayments:

- Prevailing mortgage rates.
  - Spread of current vs. original mortgage rates.
  - Mortgage rate path (refinancing burnout).
  - Level of mortgage rates.
- Underlying mortgage characteristics.
- Seasonal factors.
- General economic activity.

Contraction risk : occurs as rates fall, prepayments rise, average life falls.

Extension risk : occurs as rates rise, prepayments fall (slow), average life rises.

CMO Prepayment Risk

PAC tranche : receives cash flows according to a predetermined schedule.

Support tranche : receives prepayments in excess of PAC upper rate, diverts cash to PAC tranche if prepayments are slower than expected.

Certainty of PAC bond cash flow comes at the expense of increased risk to the support tranches.

Stripped MBSs

PO strips : receives principal payments; sold at discount from par; increases in value as prepayments increase; inverse relationship with interest rates.

IO strips : receives interest payments; investors want prepayments to be slow; positive relationship with interest rates.

INVESTMENT AND RISK MANAGEMENT

Risk and Return of a Portfolio

Expected return :

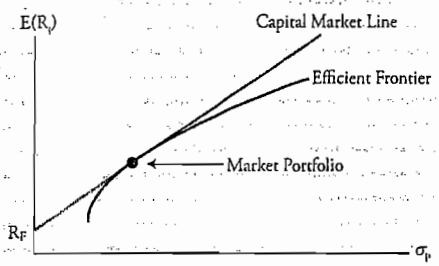
$$E(R_p) = \sum_{i=1}^{N} W_i E(R_i)$$

Variance for 2-asset portfolio :

$$\sigma^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{1,2}\sigma_1\sigma_2$$

The lower the correlation, the greater the benefits from diversification.

Capital Market Theory



Security Market Line (SML)

Investors should only be compensated for risk relative to the market. Unsystematic risk is diversified away; investors are compensated for systematic risk. The equation of the SML is the CAPM, which is a return/systematic risk equilibrium relationship.

$$\text{CAPM: } E(R_i) = R_F + \beta_i\left[E(R_{mkt}) - R_F\right]$$

Forms of Market Efficiency

Weak form : all past patterns are incorporated into market prices.
Semi-strong form : all public information is incorporated into market prices.
Strong form : all public and private information is incorporated into market prices.

Measures of Risk-Adjusted Performance

$$\text{Treynor measure} = \frac{E(R_p) - R_F}{\beta_p}$$

$$\text{Sharpe measure} = \frac{E(R_p) - R_F}{\sigma_p}$$

$$\text{Jensen's Alpha} = \alpha_p = E(R_p) - R_F - \left[E(R_M) - R_F\right]\beta_p$$

Tracking error, σ_{e_p}, is the standard deviation of the difference between returns of the portfolio and returns of the benchmark.

$$\text{information ratio} = \left[\frac{E(R_P) - E(R_B)}{\sigma_{e_p}}\right] = \frac{\alpha_p}{\sigma_{e_p}}$$

Hedge Fund Strategies

- *Long/short equity* : long an equity portfolio, and simultaneously short other equity securities.
- *Equity market neutral* : long and short stock positions of related companies.
- *Equity market timing* : screen stocks using technical trading rules to find short-term profit opportunities.
- *Short-selling* : stocks are borrowed and then sold in the public market.
- *Convertible arbitrage* : convertible securities are purchased, and a percentage of the issuer's stock is sold short.
- *Fixed-income arbitrage* : the spread between similar fixed-income securities and their derivatives is exploited.
- *Volatility arbitrage* : positions in options on fixed-income securities that have mispriced volatility.
- *Capital structure arbitrage* : capitalize on discrepancies observed between debt and equity securities.
- *Event-driven* : a portfolio of undervalued or overvalued securities with respect to company-specific events.
- *Merger arbitrage* : long and short positions in companies that are parties to corporate transaction.
- *Distressed securities* : long position in the financial securities of a financially troubled company.
- *Regulation D* : long position in privately placed unregistered securities issued by a publicly traded small company.
- *Global macro* : based around expected changes in global capital market prices.
- *Managed futures* : establish long and short futures positions on commodities and financial securities.

Hedge Fund Diversification Classifications

- *Return enhancer* : high return, high correlation.
- *Risk reducer* : low return, low correlation.
- *Total diversifier* : high return, low correlation.
- *Pure diversifier* : low or negative returns, high negative correlation.

Hedge Fund Index Problems

- Survivorship bias.
- Backfilling.
- Asset weighting.
- Selection bias.
- Autocorrelation.
- Time-period bias.

Convergence Trading

Mean reversion strategy: long undervalued asset, short overvalued asset with anticipation of narrowing yield spreads.

Forms of Leverage

Balance sheet leverage : using credit to finance positions.
Instrument leverage : riskiness inherent in a hedge fund's investments.

Transparency

There is a tradeoff with transparency for hedge fund investors. Investors want enough information to manage their risk; however, disclosing details about portfolio positions to trading partners/imitators may impair the fund's ability to acquire or liquidate positions.

Attributes of Transparency

- *Content* : quality and reliability of data.
- *Granularity* : level of detail about data.
- *Frequency* : timing and regularity of disclosures.
- *Delay* : time lag between period covered and disclosure.

Hedge Fund Style Drift

- Change in risk factor exposures of the fund.
- Change in overall risk of the fund (primarily through leverage).

Hedge Fund Style Drift Red Flags

- Poor style or manager performance.
- Excessive cash inflows.
- Recent losses.
- Personnel change.
- Regulatory change.

money; close to 0 when option is deep in- or out-of-the-money.
Rho : sensitivity of option's price to changes in the risk-free rate; largest for in-the-money options.

Delta-Neutral Hedging

- To completely hedge a long stock/short call position, purchase shares of stock equal to delta × number of options sold.
- Only appropriate for small changes in the value of the underlying asset.
- Gamma can correct hedging error by protecting against large movements in asset price.
- Gamma-neutral positions are created by matching portfolio gamma with an offsetting option position.

Exotic Options

Compound option : option on another option.
Chooser option : owner chooses whether option is a call or put after initiation.
Barrier option : payoff and existence depend on price reaching a certain barrier level.
Binary option : pay either nothing or a fixed amount.
Lookback option : payoff depends on the maximum (call) or minimum (put) value of the underlying asset over the life of the option.
Shout option : owner receives intrinsic value of option at shout date or expiration, whichever is greater.
Asian option : payoff depends on average of the underlying asset price over the life of the option; less volatile than standard option.

CREDIT RISK MANAGEMENT

Contractually Promised Gross Loan Return

$$1 + k = 1 + \frac{f + (L + m)}{1 - \left[b(1 - R)\right]}$$

Compensating balance: proportion of the loan that borrowers must keep on deposit with bank; raises the effective cost of the loan.

...CONTINUED

Probability of Default

$$\text{probability of repayment, p} = \frac{(1+i)}{(1+k)}$$

$$\text{probability of default} = 1 - p$$

$$\text{cumulative probability of default} = C_p = 1 - (p_1 \times p_2 \times \ldots \times p_n)$$

Concentration Limits

Concentration limit : maximum permitted loan amount to any individual borrower in a given sector.

$$\text{concentration limit} = \text{maximum loss as a percent of capital} \times \frac{1}{\text{loss rate}}$$

Loan Sale Terminology

Without recourse : loan is removed from bank's balance sheet; purchaser bears all credit risk.
With recourse : purchaser has the right to sell loan back to originating bank; loan is kept on bank's balance sheet as a contingent credit liability.
Participation basis : buyer buys a portion of (participates in) a larger loan; agreement between loan seller and borrower remains in place—buyer has limited control over borrower.
Assignment basis : buyer has direct claim on borrower; over 90% of loan sales in U.S. are done on an assignment basis.

Securitization

The process of issuing securities against an asset pool. Involves some, if not all, of the following participants: originator, sponsor, asset purchaser, trustee, custodian, servicer, structuring agent, underwriter, rating agency, law firms, regulatory agencies, and risk finance counterparties.

Derivative Exposure

- *Current exposure* : cost of replacing contract under current market conditions.
- *Potential exposure* : amount based on future interest rates.
- *Peak exposure* : replacement cost under worst-case scenario (bankruptcy).
- *Right way exposure* : positively correlated with counterparty's credit quality; reduces expected credit losses.
- *Wrong way exposure* : negatively correlated with counterparty's credit quality; increases expected credit losses.

Mean Loss Rates

Mean loss rate : expected loss on a risky bond due to default risk.

$$\text{mean loss rate} = PD(1 - \text{recovery rate})$$

Risk-neutral mean loss rate : where investors act as if they are risk-neutral because the rate includes an artificially high mean loss rate that reflects a risk premia for accepting the higher default risk; key input for credit risk pricing applications.

Suboptimal Loan Recovery Rates

Four factors that lead to suboptimal behavior by firms with respect to their borrowing include:

- Debt structure.
- Bargaining power of the debtor.
- Excessive rights or control by senior debtholders.
- Presence of one or more house banks.

The Merton Model

Payment to debtholders:

$$D_M - \max(D_M - V_M, 0)$$

Payment to shareholders:

$$\max(V_M - D_M, 0)$$

Firm Capital Structure in Context of Option Pricing

- Equity is similar to a long call option on the value of a firm's assets where face value of debt is the strike price of the option.
- Debt is similar to a risk-free bond and short put option on the value of a firm's assets where face value of debt is the strike price of the option.

Credit Scoring Models

- *Linear discriminant analysis* : segregates a larger group into subgroups.
- *Parametric discrimination* : uses a score function.
- K-*nearest neighbor* : categorizes new entrant by how closely it resembles other firms.
- *Support vector machines* : creates an equation that divides larger group into two subgroups.

Credit Analysis Decision Rules

- *Minimum error* : uses Bayes' theorem.
- *Minimum risk* : minimizes the probability of misclassification.
- *Neyman-Pearson* : uses the statistical concept of Type I and Type II errors.
- *Minimax* : minimizes the maximum error or risk.

Vulnerable Options

Option with default risk; holder receives promised payment only if seller of the option is able to make the payment.

$$\text{value of vulnerable option} = (1 - p)c + pzc$$

Credit Risk Portfolio Models

CreditMetrics : computes the probability of default, loss given default, and correlations.
KMV's Portfolio Manager : multivariate normal distribution; has only one factor and only computes default losses.
Portfolio Risk Tracker : dynamic analysis (e.g., it simulates risk factors for periods within the chosen horizon).
CreditPortfolioView : econometric model; drives the default rates of industrial sectors.
CreditRisk+ : actuarial approach; predicts the proportion of obligors that will default in a particular sector and in the portfolio of interest.

Portfolio Risk Indicators

- *Expected loss* : based on probability, exposure, and loss given default.

$$E(L_i) = EAD_i \times PD_i \times LGD_i$$

- *Unexpected loss* : standard deviation of the losses for a given portfolio.

$$\sqrt{\left(E\left[L_p - E(L_p)\right]^2\right)}$$

OPERATIONAL AND INTEGRA

BIS Definition of Operational Risk

"The risk of losses due to inadequate or failed processes, persons, and systems that cannot protect a firm from outside events."
The BIS definition focuses on the impact of operational losses, but it excludes credit and market risks.

Classifications of Operational Risk

High-frequency, low-severity (HFLS) risks occur frequently but result in small losses.
Low-frequency, high-severity (LFHS) risks are the greatest area of concern for operational risk managers. Because they are rare, there is little available data to analyze such risks, and their cost to the firm could be catastrophic.

Top-Down and Bottom-Up Models

Top-down model : examines aggregate impact of operational failures, macro view; relies on historical data.

- Advantages: simple to use, not data-intensive.
- Disadvantages: does not distinguish between HFLS events and LFHS events; cannot diagnose specific areas of weakness; backward looking.

Bottom-up model : analyzes risk in individual processes.

- Advantages: distinguishes between HFLS events

- *VAR* : value of loss, exceeded only a certain percent of the time.
- *Economic capital* : amount of capital needed as a buffer to avoid insolvency.
- *Expected shortfall* : expected value of the loss given that loss > VAR.

Credit Derivatives

Credit default swap : like insurance; party selling the protection receives a fee, pays based on swap's notional amount in the case of default.
First-to-default put : reduces credit risk on a basket of loans; only first loan in the basket to default triggers payment; if more than one loan defaults, swap only pays on first default.
Total return swap : total return on an asset (bond) is exchanged for a fixed (or variable) payment; total return receiver gets any appreciation (capital gains and cash flows), pays any depreciation; payments take place whether or not a credit event occurs.
Buyer of credit derivative exchanges credit risk of issuer defaulting for the combined risk of the issuer and the derivative counterparty.

Credit Spread Options

$$\text{put payoff}_t = NP \times \text{duration} \times \max[\text{credit spread}_t - \text{strike spread}, 0]$$

$$\text{call payoff}_t = NP \times \text{duration} \times \max[\text{strike spread} - \text{credit spread}_t, 0]$$

Credit at Risk (CAR)

Measures the potential loss in a bond investment from a change in the asset's credit quality:

$$CAR_t = NP \times \sigma \times z \times \sqrt{t}$$

FED RISK MANAGEMENT

and LFHS events; can diagnose weaknesses in procedures and suggest corrections; forward looking.
- Disadvantages: complex and data intensive.

Methods for Hedging Operational Risk
- Insurance.
- Self-insurance.
- Derivative securities.

Catastrophe Options and Bonds
Cat options: Publicly traded; payoffs linked to index (i.e., underwriting losses in the insurance industry); spread option that has limited upside.
Cat bonds: Bond contracts with embedded options that can be triggered by internal events, external events, or the value of an index.

Economies of Scale and Scope
Economies of scale: average costs of production decline as an institution grows.
Economies of scope: synergy of producing two products jointly rather than each one independently; the cost of producing two or more products together is less than the sum of the costs of producing each product individually.

Daylight Overdraft Risk
During the day, banks are allowed to have negative intraday balances on their required reserve accounts with the Fed. If a bank failure occurs while the bank has a negative balance, the Fed must absorb the overdraft, which could cause negative effects throughout the financial system.

Loss Data
Operational risk loss data can be classified into one of five categories:
1. *External.* out of the direct control of the organization.
2. *People.* Loss as the direct result of human actions.
3. *Process.* Loss from execution, transaction, processing, or producing a good or service.
4. *Relationship.* Risk related to relationships with internal groups or external relationships.
5. *Technology.* Losses related to technology issues.

Model Risk
The risk associated with using financial models to simulate complex relationships. Sources of model risk include incorrect model application, implementation risk, calibration errors, programming errors, and data problems.

Case Studies
Metallgesellschaft: short-term futures contracts used to hedge long-term exposure in the petroleum markets; stack-and-roll hedging strategy; marking to market on futures caused huge cash flow problems.
Sumitomo: trader attempted to corner the copper market by buying large quantities of physical copper and long futures positions; copper prices plunged, causing huge losses; lesson is the lack of operational and risk controls that allowed this scheme to go undetected.
Long-Term Capital Management: hedge fund that used relative value strategies with enormous amounts of leverage; when Russia defaulted on its debt in 1998, the increase in yield spreads caused huge losses and enormous cash flow problems from realizing marking-to-market losses; lessons include lack of diversification, model risk, leverage, and funding and trading liquidity risks.
Barings: rogue trader, Nick Leeson, took speculative derivative positions (Nikkei 225 futures) in an attempt to cover trading losses; Leeson had dual responsibilities of trading and supervising settlement operations, allowing him to hide trading losses; lessons include separation of duties and management oversight.
Amaranth Advisors: lost about two-thirds of its value. Trades in natural gas futures and options appear to be the most likely cause. The larger than expected loss was probably caused by liquidity problems. The case points out how risk measures should include liquidity risk.

Enterprise Risk Management
The framework of ERM is a 4-step process:
Step 1: Determine the firm's risk appetite.
Step 2: Estimate the amount of capital needed to support the desired level of risk.
Step 3: Determine the optimal combination of capital and risk that achieves the target credit rating.
Step 4: Decentralize the management of risk.

CAPITAL ALLOCATION

Economic Capital
Designed to provide a cushion against *unexpected losses* at a specified confidence level. Note that economic capital > regulatory capital.

Risk-Adjusted Return on Capital
Defined as risk-adjusted return/risk-adjusted capital.

$$\text{RAROC} = \frac{\text{revenues} - \begin{pmatrix}\text{expected}\\ \text{loss}\end{pmatrix} - \text{expenses} + \begin{pmatrix}\text{return on}\\ \text{economic capital}\end{pmatrix} \pm \begin{matrix}\text{transfer}\\ \text{price}\end{matrix}}{\text{economic capital}}$$

The ARAROC approach addresses RAROC's flaw of assuming the probability of default is constant.

$$\text{ARAROC} = \frac{(\text{RAROC} - R_F)}{\beta_E},$$

Allocating Economic Capital
- *Stand-alone method*: external data in a value-based approach.
- *Scaling method*: internal data in a RAROC approach.
- *Internal betas method*: internal and external data in a business-unit, return-based approach.
- *Marginal capital method*: internal data in a business-unit, value-based approach.
- *APT method*: external data in a business-unit, value-based approach.
- *Fair value method*: internal and external data in a business-unit, return-based approach.

Justifications for Banking Regulation
- Protect bank depositor from a loss in bankruptcy.
- Provide stability for transactions.
- Avoid domino effects on the banking system.
- Maintain stability in the economy.

Basel II: 3 Pillars
1. Minimum capital requirements.
2. Supervisory review process.
3. Market discipline.

Basel II: Forms of Capital
Tier 1: shareholder's equity, retained earnings; nonredeemable, noncumulative preferred stock.
Tier 2: cumulative preferred stock, gains on LT investments, loan loss reserves.
Tier 3: short-term subordinated debt, can only be used to offset market risks.

Basel II: Capital Requirements
Standardized approach: based on external credit rating assessments.
Internal ratings based (IRB) approaches:
- Foundation: bank estimates PD.
- Advanced: bank estimates PD, LGD, EAD, and M.

Basel II: Operational Risk
- *Basic indicator approach*: capital charge measured on a firmwide basis.
- *Standardized approach*: banks divide activities among business lines; capital charge = sum for each business line.
- *Advanced measurement approach*: banks use their own methodologies for assessing operational risk.

Capital Management
The problem of capital adequacy for financial conglomerates can be resolved by determining:
- Portfolio's single-factor risk.
- The business unit's cross-risk factors.
- Risk factors across the business units at the holding company level.

ISBN 1-60373-013-3
9 781603 730136
U.S. $29.00