EXAMPLE 5.1: TIME SCALING

Consider a portfolio with a one-day VAR of $1 million. Assume that the market is trending with an autocorrelation of 0.1. Under this scenario, what would you expect the two-day VAR to be?

a. $2 million
b. $1.414 million
c. $1.483 million
d. $1.449 million

EXAMPLE 5.2: INDEPENDENCE

A fundamental assumption of the random walk hypothesis of market returns is that returns from one time period to the next are statistically independent. This assumption implies

a. Returns from one time period to the next can never be equal.
b. Returns from one time period to the next are uncorrelated.
c. Knowledge of the returns from one time period does not help in predicting returns from the next time period.
d. Both b. and c. are true.

EXAMPLE 5.3: FRM EXAM 2002—QUESTION 3

Consider a stock with daily returns that follow a random walk. The annualized volatility is 34%. Estimate the weekly volatility of this stock assuming that the year has 52 weeks.

a. 6.80%
b. 5.83%
c. 4.85%
d. 4.71%

EXAMPLE 5.4: FRM EXAM 2002—QUESTION 2

Assume we calculate a one-week VAR for a natural gas position by rescaling the daily VAR using the square root of time rule. Let us now assume that we determine the *true* gas price process to be mean reverting and recalculate the VAR. Which of the following statements is true?

a. The recalculated VAR will be less than the original VAR.
b. The recalculated VAR will be equal to the original VAR.
c. The recalculated VAR will be greater than the original VAR.
d. There is no necessary relationship between the recalculated VAR and the original VAR.

EXAMPLE 5.5: FRM EXAM 2004—QUESTION 39

Consider a portfolio with 40% invested in asset X and 60% invested in asset Y. The mean and variance of return on X are 0 and 25 respectively. The mean and variance of return on Y are 1 and 121 respectively. The correlation coefficient between X and Y is 0.3. What is the nearest value for portfolio volatility?

a. 9.51
b. 8.60
c. 13.38
d. 7.45

EXAMPLE 5.6: FRM EXAM 2009—QUESTION 2-13

Suppose σ_t^2 is the estimated variance at time t and u_t is the realized return at t. Which of the following GARCH(1,1) models will take the longest time to revert to its mean?

a. $\sigma_t^2 = 0.04 + 0.02u_{t-1}^2 + 0.92\sigma_{t-1}^2$
b. $\sigma_t^2 = 0.02 + 0.04u_{t-1}^2 + 0.94\sigma_{t-1}^2$
c. $\sigma_t^2 = 0.03 + 0.02u_{t-1}^2 + 0.95\sigma_{t-1}^2$
d. $\sigma_t^2 = 0.03 + 0.03u_{t-1}^2 + 0.93\sigma_{t-1}^2$

EXAMPLE 5.7: FRM EXAM 2006—QUESTION 132

Assume you are using a GARCH model to forecast volatility that you use to calculate the one-day VAR. If volatility is mean reverting, what can you say about the T-day VAR?

a. It is less than the $\sqrt{T}\times$ one-day VAR.
b. It is equal to $\sqrt{T}\times$ one-day VAR.
c. It is greater than the $\sqrt{T}\times$ one-day VAR.
d. It could be greater or less than the $\sqrt{T}\times$ one-day VAR.

EXAMPLE 5.8: FRM EXAM 2007—QUESTION 34

A risk manager estimates daily variance h_t using a GARCH model on daily returns r_t: $h_t = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta h_{t-1}$, with $\alpha_0 = 0.005$, $\alpha_1 = 0.04$, $\beta = 0.94$. The long-run *annualized* volatility is approximately

a. 13.54%
b. 7.94%
c. 72.72%
d. 25.00%

EXAMPLE 5.9: FRM EXAM 2009—QUESTION 2-17

Which of the following statements is *incorrect* regarding the volatility term structure predicted by a GARCH(1,1) model: $\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \beta\sigma_{t-1}^2$, where $\alpha + \beta < 1$?

a. When the current volatility estimate is below the long-run average volatility, this GARCH model estimates an upward-sloping volatility term structure.
b. When the current volatility estimate is above the long-run average volatility, this GARCH model estimates a downward-sloping volatility term structure.
c. Assuming the long-run estimated variance remains unchanged as the GARCH parameters α and β increase, the volatility term structure predicted by this GARCH model reverts to the long-run estimated variance more slowly.
d. Assuming the long-run estimated variance remains unchanged as the GARCH parameters α and β increase, the volatility term structure predicted by this GARCH model reverts to the long-run estimated variance faster.

EXAMPLE 5.10: FRM EXAM 2007—QUESTION 46

A bank uses the exponentially weighted moving average (EWMA) technique with λ of 0.9 to model the daily volatility of a security. The current estimate of the daily volatility is 1.5%. The closing price of the security is USD 20 yesterday and USD 18 today. Using continuously compounded returns, what is the updated estimate of the volatility?

a. 3.62%
b. 1.31%
c. 2.96%
d. 5.44%

EXAMPLE 5.11: FRM EXAM 2006—QUESTION 40

Using a daily RiskMetrics EWMA model with a decay factor $\lambda = 0.95$ to develop a forecast of the conditional variance, which weight will be applied to the return that is four days old?

a. 0.000
b. 0.043
c. 0.048
d. 0.950

EXAMPLE 5.12: EFFECT OF WEIGHTS ON OBSERVATIONS

Until January 1999 the historical volatility for the Brazilian real versus the U.S. dollar had been very small for several years. On January 13, Brazil abandoned the defense of the currency peg. Using the data from the close of business on January 13, which of the following methods for calculating volatility would have shown the greatest jump in measured historical volatility?

a. 250-day equal weight
b. Exponentially weighted with a daily decay factor of 0.94
c. 60-day equal weight
d. All of the above

EXAMPLE 5.13: FRM EXAM 2008—QUESTION 1-8

Which of the following four statements on models for estimating volatility is *incorrect*?

a. In the EWMA model, some positive weight is assigned to the long-run average variance rate.
b. In the EWMA model, the weights assigned to observations decrease exponentially as the observations become older.
c. In the GARCH(1,1) model, a positive weight is estimated for the long-run average variance rate.
d. In the GARCH(1,1) model, the weights estimated for observations decrease exponentially as the observations become older.

EXAMPLE 5.14: FRM EXAM 2009—QUESTION 2-16

Assume that an asset's daily return is normally distributed with zero mean. Suppose you have historical return data, $u_1, u_2, \ldots, u_m$ and that you want to use the maximum likelihood method to estimate the parameters of a EWMA volatility model. To do this, you define $v_i = \sigma_i^2$ as the variance estimated by the EWMA model on day i, so that the likelihood that these m observations occurred is given by: $\Pi_{i=1}^{m}[\frac{1}{\sqrt{2\pi v_i}}\exp[-u_i^2/(2v_i)]$. To maximize the likelihood that these m observations occurred, you must:

a. Find the value of λ that minimizes: $\sum_{i=1}^{m}[-\ln(v_i) - u_i^2/(2v_i)]$
b. Find the value of λ that maximizes: $\sum_{i=1}^{m}[-\ln(v_i) - u_i^2/(2v_i)]$
c. Find the value of λ that minimizes: $-m\ln(v_i) - \sum_{i=1}^{m}[u_i^2/(2v_i)]$
d. Find the value of λ that maximizes: $-m\ln(v_i) - \sum_{i=1}^{m}[u_i^2/(2v_i)]$

5.6 ANSWERS TO CHAPTER EXAMPLES

Example 5.1: Time Scaling

c. Knowing that the variance is $V(2\text{-day}) = V(1\text{-day})\,[2 + 2\rho]$, we find VAR(2-day) = VAR(1-day) $\sqrt{2 + 2\rho} = \$1\sqrt{2 + 0.2} = \$1.483$, assuming the same distribution for the different horizons.

Example 5.2: Independence

d. The term *efficient markets* implies that the distribution of future returns does not depend on past returns. Hence, returns cannot be correlated. It could happen, however, that return distributions are independent but that, just by chance, two successive returns are equal.

Example 5.3: FRM Exam 2002—Question 3

d. Assuming a random walk, we can use the square root of time rule. The weekly volatility is then $34\% \times 1/\sqrt{52} = 4.71\%$.

Example 5.4: FRM Exam 2002—Question 2

a. With mean reversion, the volatility grows more slowly than the square root of time.

Example 5.5: FRM Exam 2004—Question 39

d. The variance of the portfolio is given by $\sigma_p^2 = (0.4)^2 25 + (0.6)^2 121 + 2(0.4)(0.6)0.3\sqrt{25 \times 121} = 55.48$. Hence the volatility is 7.45.

Example 5.6: FRM Exam 2009—Question 2-13

b. The persistence $(\alpha_1 + \beta)$ is, respectively, 0.94, 0.98, 0.97, and 0.96. Hence the model with the highest persistence will take the longest time to revert to the mean.

Example 5.7: FRM Exam 2006—Question 132

d. If the initial volatility were equal to the long-run volatility, then the T-day VAR could be computed using the square root of time rule, assuming normal distributions. If the starting volatility were higher, then the T-day VAR should be less than the $\sqrt{T}\times$ one-day VAR. Conversely, if the starting volatility were lower, then the T-day VAR should be more than the long-run value. However, the question does not indicate the starting point. Hence, answer d. is correct.

Example 5.8: FRM Exam 2007—Question 34

b. The long-run mean variance is $h = \alpha_0/(1 - \alpha_1 - \beta) = 0.005/(1 - 0.04 - 0.94) = 0.25$. Taking the square root, this gives 0.5 for daily volatility. Multiplying by $\sqrt{252}$, we have an annualized volatility of 7.937%.

Example 5.9: FRM Exam 2009—Question 2-17

d. The GARCH model has mean reversion in the conditional volatility, so statements a. and b. are correct. When σ_t is lower than the long-run average, the volatility structure goes up. Higher persistence $\alpha + \beta$ means that mean reversion is slower, so statement c. is correct.

Example 5.10: FRM Exam 2007—Question 46

a. The log return is $\ln(18/20) = -10.54\%$. The new variance forecasts is $h = 0.90 \times (1.5^2) + (1 - 0.90) \times 10.54^2 = 0.001313$, or taking the square root, 3.62%.

Example 5.11: FRM Exam 2006—Question 40

b. The weight of the last day is $(1 - 0.95) = 0.050$. For the day before, this is 0.05×0.95, and for four days ago, $0.05 \times 0.95^3 = 0.04287$.

Example 5.12: Effect of Weights on Observations

b. The EWMA model puts a weight of 0.06 on the latest observation, which is higher than the weight of $(1/60) = 0.0167$ for the 60-day MA and $(1/250) = 0.004$ for the 250-day MA.

Example 5.13: FRM Exam 2008—Question 1-8

a. The GARCH model has a finite unconditional variance, so statement c. is correct. In contrast, because $\alpha_1 + \beta$ sum to 1, the EWMA model has undefined long-run average variance. In both models weights decline exponentially with time.

Example 5.14: FRM Exam 2009—Question 2-16

b. The optimal parameter must maximize (not minimize) the likelihood function. Otherwise, the log-likelihood function is the log of the product, which is the sum of the logs. This gives, up to a constant, $\sum_{i=1}^{m}[-\ln(v_i) - u_i^2/(2v_i)]$, and there is no way to take the first term outside the summation because it depends on i. So, answers c. and d. are incorrect.