EXAMPLE 6.1: FRM EXAM 2002—QUESTION 48

An investor buys a Treasury bill maturing in one month for $987. On the maturity date the investor collects $1,000. Calculate effective annual rate (EAR).

a. 17.0%
b. 15.8%
c. 13.0%
d. 11.6%

EXAMPLE 6.2: FRM EXAM 2009—QUESTION 4-9

Lisa Smith, the treasurer of Bank AAA, has $100 million to invest for one year. She has identified three alternative one-year certificates of deposit (CDs), with different compounding periods and annual rates. CD1: monthly, 7.82%; CD2: quarterly, 8.00%; CD3: semiannually, 8.05%; and CD4: continuous, 7.95%. Which CD has the highest effective annual rate (EAR)?

a. CD1
b. CD2
c. CD3
d. CD4

EXAMPLE 6.3: FRM EXAM 2002—QUESTION 51

Consider a savings account that pays an annual interest rate of 8%. Calculate the amount of time it would take to double your money. Round to the nearest year.

a. 7 years
b. 8 years
c. 9 years
d. 10 years

EXAMPLE 6.4: FRM EXAM 2009—QUESTION 4-8

A five-year corporate bond paying an annual coupon of 8% is sold at a price reflecting a yield to maturity of 6%. One year passes and the interest rates remain unchanged. Assuming a flat term structure and holding all other factors constant, the bond's price during this period will have

a. Increased
b. Decreased
c. Remained constant
d. Cannot be determined with the data given

EXAMPLE 6.5: FRM EXAM 2006—QUESTION 75

A zero-coupon bond with a maturity of 10 years has an annual effective yield of 10%. What is the closest value for its modified duration?

a. 9 years
b. 10 years
c. 99 years
d. 100 years

EXAMPLE 6.6: FRM EXAM 2007—QUESTION 115

A portfolio manager has a bond position worth USD 100 million. The position has a modified duration of eight years and a convexity of 150 years. Assume that the term structure is flat. By how much does the value of the position change if interest rates increase by 25 basis points?

a. USD −2,046,875
b. USD −2,187,500
c. USD −1,953,125
d. USD −1,906,250

EXAMPLE 6.7: FRM EXAM 2009—QUESTION 4-15

A portfolio manager uses her valuation model to estimate the value of a bond portfolio at USD 125.482 million. The term structure is flat. Using the same model, she estimates that the value of the portfolio would increase to USD 127.723 million if all interest rates fell by 30bp and would decrease to USD 122.164 million if all interest rates rose by 30bp. Using these estimates, the effective duration of the bond portfolio is closest to:

a. 8.38
b. 16.76
c. 7.38
d. 14.77

EXAMPLE 6.8: FRM EXAM 2003—QUESTION 13

Suppose the face value of a three-year option-free bond is USD 1,000 and the annual coupon is 10%. The current yield to maturity is 5%. What is the modified duration of this bond?

a. 2.62
b. 2.85
c. 3.00
d. 2.75

EXAMPLE 6.9: FRM EXAM 2002—QUESTION 118

A Treasury bond has a coupon rate of 6% per annum (the coupons are paid semiannually) and a semiannually compounded yield of 4% per annum. The bond matures in 18 months and the next coupon will be paid 6 months from now. Which number of years is closest to the bond's Macaulay duration?

a. 1.023 years
b. 1.457 years
c. 1.500 years
d. 2.915 years

EXAMPLE 6.10: DURATION AND COUPON

A and B are two perpetual bonds; that is, their maturities are infinite. A has a coupon of 4% and B has a coupon of 8%. Assuming that both are trading at the same yield, what can be said about the duration of these bonds?

a. The duration of A is greater than the duration of B.
b. The duration of A is less than the duration of B.
c. A and B both have the same duration.
d. None of the above.

EXAMPLE 6.11: FRM EXAM 2004—QUESTION 16

A manager wants to swap a bond for a bond with the same price but higher duration. Which of the following bond characteristics would be associated with a higher duration?

I. A higher coupon rate
II. More frequent coupon payments
III. A longer term to maturity
IV. A lower yield

a. I, II, and III
b. II, III, and IV
c. III and IV
d. I and II

EXAMPLE 6.12: FRM EXAM 2001—QUESTION 104

When the maturity of a plain coupon bond increases, its duration increases

a. Indefinitely and regularly
b. Up to a certain level
c. Indefinitely and progressively
d. In a way dependent on the bond being priced above or below par

EXAMPLE 6.13: FRM EXAM 2000—QUESTION 106

Consider the following bonds:

Bond Number	Maturity (Years)	Coupon Rate	Frequency	Yield (Annual)
1	10	6%	1	6%
2	10	6%	2	6%
3	10	0%	1	6%
4	10	6%	1	5%
5	9	6%	1	6%

How would you rank the bonds from the shortest to longest duration?

a. 5-2-1-4-3
b. 6-2-3-4-5
c. 5-4-3-1-2
d. 2-4-5-1-3

EXAMPLE 6.14: FRM EXAM 2000—QUESTION 110

Which of the following statements is/are *true*?

I. The convexity of a 10-year zero-coupon bond is higher than the convexity of a 10-year 6% bond.
II. The convexity of a 10-year zero-coupon bond is higher than the convexity of a 6% bond with a duration of 10 years.
III. Convexity grows proportionately with the maturity of the bond.
IV. Convexity is always positive for all types of bonds.
V. Convexity is always positive for straight bonds.

a. I only
b. I and II only
c. I and V only
d. II, III, and V only

EXAMPLE 6.15: FRM EXAM 2002—QUESTION 57

A bond portfolio has the following composition:

1. Portfolio A: price \$90,000, modified duration 2.5, long position in 8 bonds
2. Portfolio B: price \$110,000, modified duration 3, short position in 6 bonds
3. Portfolio C: price \$120,000, modified duration 3.3, long position in 12 bonds

All interest rates are 10%. If the rates rise by 25 basis points, then the bond portfolio value will decrease by

a. \$11,430
b. \$21,330
c. \$12,573
d. \$23,463

EXAMPLE 6.16: FRM EXAM 2006—QUESTION 61

Consider the following portfolio of bonds (par amounts are in millions of USD).

Bond	Price	Par Amount Held	Modified Duration
A	101.43	3	2.36
B	84.89	5	4.13
C	121.87	8	6.27

What is the value of the portfolio's DV01 (dollar value of 1 basis point)?

a. \$8,019
b. \$8,294
c. \$8,584
d. \$8,813

EXAMPLE 6.17: FRM EXAM 2008—QUESTION 2-33

Which of the following statements is *correct* regarding the effects of interest rate shift on fixed-income portfolios with similar durations?

a. A barbell portfolio has greater convexity than a bullet portfolio because convexity increases linearly with maturity.
b. A barbell portfolio has greater convexity than a bullet portfolio because convexity increases with the square of maturity.
c. A barbell portfolio has lower convexity than a bullet portfolio because convexity increases linearly with maturity.
d. A barbell portfolio has lower convexity than a bullet portfolio because convexity increases with the square of maturity.

6.6 ANSWERS TO CHAPTER EXAMPLES

Example 6.1: FRM Exam 2002—Question 48

a. The EAR is defined by $\text{FV}/\text{PV} = (1 + \text{EAR})^T$. So $\text{EAR} = (\text{FV}/\text{PV})^{1/T} - 1$. Here, $T = 1/12$. So, $\text{EAR} = (1{,}000/987)^{12} - 1 = 17.0\%$.

Example 6.2: FRM Exam 2009—Question 4-9

d. A dollar initially invested will grow to (CD1) $(1 + 7.82\%/12)^{12} = 1.08107$, (CD2) $(1 + 8.00\%/4)^4 = 1.08243$, (CD3) $(1 + 8.05\%/2)^2 = 1.08212$, (CD4) $\exp(7.95\%) = 1.08275$. Hence, CD4 gives the highest final amount and EAR.

Example 6.3: FRM Exam 2002—Question 51

c. The time T relates the current and future values such that $\text{FV}/\text{PV} = 2 = (1 + 8\%)^T$. Taking logs of both sides, this gives $T = \ln(2)/\ln(1.08) = 9.006$.

Example 6.4: FRM Exam 2009—Question 4-8

b. Because the coupon is greater than the yield, the bond must be selling at a premium, or current price greater than the face value. If yields do not change, the bond price will converge to the face value. Given that it starts higher, it must decrease.

Example 6.5: FRM Exam 2006—Question 75

a. Without doing any computation, the Macaulay duration must be 10 years because this is a zero-coupon bond. With annual compounding, modified duration is $D^* = 10/(1 + 10\%)$, or close to 9 years.

Example 6.6: FRM Exam 2007—Question 115

c. The change in price is given by $\Delta P = -[D^* \times P](\Delta y) + \frac{1}{2}[C \times P](\Delta y)^2$ $= -[8 \times 100](0.0025) + 0.5[150 \times 100](0.0025)^2 = -2.000000 + 0.046875 = -1.953125$.

Example 6.7: FRM Exam 2009—Question 4-15

c. By Equation (6.20), effective duration is $D^E = \frac{[P_- - P_+]}{(2P_0\Delta y)} = \frac{[127.723 - 122.164]}{(125.482 \times 0.6\%)} = 7.38$.

Example 6.8: FRM Exam 2003—Question 13

a. As in Table 6.2, we lay out the cash flows and find

Period t	Payment C_t	Yield y	$\text{PV}_t = C_t/(1+y)^t$	$t\text{PV}_t$
1	100	5.00	95.24	95.24
2	100	5.00	90.71	181.41
3	1,100	5.00	950.22	2,850.66
Sum:			1,136.16	3,127.31

Duration is then 2.75, and modified duration 2.62.

Example 6.9: FRM Exam 2002—Question 118

b. For coupon-paying bonds, Macaulay duration is slightly less than the maturity, which is 1.5 years here. So, b. would be a good guess. Otherwise, we can compute duration exactly.

Example 6.10: Duration and Coupon

c. Going back to the duration equation for the consol, Equation (6.27), we see that it does not depend on the coupon but only on the yield. Hence, the durations must be the same. The price of bond A, however, must be half that of bond B.

Example 6.11: FRM Exam 2004—Question 16

c. Higher duration is associated with physical characteristics that push payments into the future, that is, longer term, lower coupons, and less frequent coupon payments, as well as lower yields, which increase the relative weight of payments in the future.

Example 6.12: FRM Exam 2001—Question 104

b. With a fixed coupon, the duration goes up to the level of a consol with the same coupon. See Figure 6.8.

Example 6.13: FRM Exam 2000—Question 106

a. The nine-year bond (number 5) has shorter duration because the maturity is shortest, at nine years, among comparable bonds. Next, we have to decide between bonds 1 and 2, which differ only in the payment frequency. The semiannual bond (number 2) has a first payment in six months and has shorter duration than the annual bond. Next, we have to decide between bonds 1 and 4, which differ only in the yield. With lower yield, the cash flows further in the future have a higher weight, so that bond 4 has greater duration. Finally, the zero-coupon bond has the longest duration. So, the order is 5-2-1-4-3.

Example 6.14: FRM Exam 2000—Question 110

c. Because convexity is proportional to the square of time to payment, the convexity of a bond is mainly driven by the cash flows far into the future. Answer I. is correct because the 10-year zero has only one cash flow, whereas the coupon bond has several others that reduce convexity. Answer II. is false because the 6% bond with 10-year duration must have cash flows much further into the future, say in 30 years, which will create greater convexity. Answer III. is false because convexity grows with the square of time. Answer IV. is false because some bonds, for example MBSs or callable bonds, can have negative convexity. Answer V. is correct because convexity must be positive for coupon-paying bonds.

Example 6.15: FRM Exam 2002—Question 57

a. The portfolio dollar duration is $D^*P = \sum x_i D_i^* P_i = +8 \times 2.5 \times \$90{,}000 - 6 \times 3.0 \times \$110{,}000 + 12 \times 3.3 \times \$120{,}000 = \$4{,}572{,}000$. The change in portfolio value is then $-(D * P)(\Delta y) = -\$4{,}572{,}000 \times 0.0025 = -\$11{,}430$.

Example 6.16: FRM Exam 2006—Question 61

c. First, the market value of each bond is obtained by multiplying the par amount by the ratio of the market price divided by 100. Next, this is multiplied by D^* to get the dollar duration DD. Summing, this gives \$85.841 million. We multiply by 1,000,000 to get dollar amounts and by 0.0001 to get the DV01, which gives \$8,584.

Bond	Price	Par	Market Value	D^*	DD
A	101.43	3	3.043	2.36	7.181
B	84.89	5	4.245	4.13	15.530
C	121.87	8	9.750	6.27	61.130
Sum					85.841

Example 6.17: FRM Exam 2008—Question 2-33

b. The statement compares two portfolios with the same duration. A barbell portfolio consists of a combination of short-term and long-term bonds. A bullet portfolio has only medium-term bonds. Because convexity is a quadratic function of time to wait for the payments, the long-term bonds create a large contribution to the convexity of the barbell portfolio, which must be higher than that of the bullet portfolio.